Phase transition and nucleon as soliton in the Nambu – Jona-Lasinio model at finite temperature and density

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We study some bulk thermodynamical characteristics, meson properties and the nucleon as a baryon number one soliton in a hot quark matter in the NJL model as well as in a hot nucleon matter in a hybrid NJL model in which the Dirac sea of quarks is combined with a Fermi sea of nucleons. In both cases, working in mean-field approximation, we find a chiral phase transition from Goldstone to Wigner phase. At finite density the chiral order parameter and the constituent quark mass have a non-monotonic temperature dependence - at finite temperatures not close to the critical one they are less affected than in the cold matter. Whereas the quark matter is rather soft against thermal fluctuations and the corresponding chiral phase transition is smooth, the nucleon matter is much stiffer and the chiral phase transition is very sharp. The thermodynamical variables show large discontinuities which is an indication for a first order phase transition. We solve the $B = 1$ solitonic sector of the NJL model in the presence of an external hot quark and nucleon medium. In the hot medium at intermediate temperature the soliton is more bound and less swelled than in the case of a cold matter. At some critical temperature, which for the nucleon matter coincides with the critical temperature for the chiral phase transition, we find no more a localized solution. According to this model scenario one should expect a sharp phase transition from the nucleon to the quark matter.

PACS numbers: 12.39.Fe, 14.20.Dh, 14.40.Aq, 11.10.Wx

I. INTRODUCTION

At some finite density and/or temperature it is generally expected a restoration of the chiral symmetry and a deconfinement, and hence a change of the structure of the hadrons immersed in a hot and dense medium. It is a topic of an increasing interest related to the evolution of the early Universe as well as to the processes in the interior of the neutron stars. Rather encouragingly, direct experimental studies of such phenomena in the relativistic heavy-ion reactions are also possible. Indeed, the lattice QCD calculations suggest phase transitions concerning both chiral symmetry restoration and color deconfinement at relatively low temperatures (of about 150 - 160 MeV) [1,2]. These low temperatures are experimentally accessible even in the existing accelerators and the corresponding heavy-ion experiments are already planned at BNL and CERN [3]. From the side of theory the low critical values mean that we have to deal with non-perturbative phenomena. Since the analytical as well as the numerical (lattice QCD) methods are not developed enough to allow fully the solution of low-energy non-perturbative, especially if the baryons are involved, it is a motivation to apply effective quark models. The Nambu - Jona-Lasinio (NJL) model [4] is the simplest purely quark model which incorporates the chiral symmetry and allows for its spontaneous breaking but it lacks confinement. It shows, however, a significant success in description of meson structure as well as of the properties of the nucleon as a non-topological soliton. Quite encouragingly, assuming that the nuclear medium can approximately be replaced by a uniform quark medium, the model describes (for review see [5]) a restoration of the chiral symmetry at finite temperature and/or density in a quantitative agreement with the lattice QCD calculations [1,2] as well as with the chiral perturbation theory [6]. Following a scenario in which the restoration of the chiral symmetry in medium is considered as a relevant mechanism for a scale change which modify the hadron structure, the NJL model seems to provide an appropriate working scheme to study the hadron properties in hot medium. Assuming that the baryon medium can be approximated by a quark medium the bulk thermodynamical variables and medium modified meson properties has been repeatedly studied [5,7,8,9,10,11,12,13]. The nucleon as a $B = 1$ soliton in a cold quark medium has been already investigated [14]. The results show that the nucleon as a soliton at finite density is less bound and

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increases in size, and at some critical value of about two times nuclear matter density the nucleon as a soliton does not exist.

In the present paper our task is to study some bulk thermodynamical characteristics and meson properties at finite temperature and density as well as the medium modification of the nucleon as a bound state of N_c quarks coupled to the polarized medium (Fermi sea) and Dirac sea. In order to respect to some extent confinement we consider two different cases, namely a medium of nucleons, which is relevant before the phase transition, as well as a quark medium. In the former case, similar to the approach of Walecka [25], the nucleons are coupled directly to the meson fields. However, the NJL model differs from the Walecka approach in the meson fields, which appear basically as quark-antiquarks excitations, and also in the way in which the chiral symmetry is dynamically broken. Such a model picture has been considered by Jaminon et al. [17] in the case of cold matter. Solving the $B = 1$ solitonic sector of the NJL model in the presence of an external hot medium in a self-consistent way with the polarization of both Fermi and Dirac sea taken into account, we study the modification of the nucleon structure due to the medium. We work in mean-field approximation, which means that the meson quantum (loop) effects are not included in our considerations. Since the meson loop effects are dominant at low temperatures and vanishing density (pions are the lightest mode) [11,12], we restrict ourselves to consider mainly the case of finite density and large enough temperatures where the nucleon and quark degrees of freedom are most relevant. This case is also of interest for the relativistic heavy-ion reactions.

II. NJL MODEL AT FINITE TEMPERATURE AND DENSITY

We use the Nambu–Jona-Lasinio (NJL) model [4] which is the simplest purely quark model describing spontaneous chiral symmetry breakdown. The SU(2)-version of the NJL lagrangian contains chirally invariant local scalar and pseudoscalar four-quark interaction:

$$\mathcal{L} = \bar{\Psi}\,(i\partial\!\!\!/ - m_0)\,\Psi + \frac{G}{2}\,[(\bar{\Psi}\Psi)^2 + (\bar{\Psi}i\vec{\tau}\gamma_5\Psi)^2\,]\;, \tag{1}$$

where Ψ is the quark field, G is the coupling constant, $\vec{\tau}$ are the Pauli matrices in the isospin space and m_0 is the current mass taken equal for both *up* and *down* quarks. Applying the well known bosonization procedure [15] the NJL model is expressed in terms of auxiliary meson fields $\sigma, \vec{\pi}$:

$$\mathcal{L} = \bar{\Psi}\;(\;i\partial\!\!\!/ - \sigma - i\,\vec{\pi}\cdot\vec{\tau}\,\gamma_5\;)\,\Psi - \frac{1}{2G}(\sigma^2 + \vec{\pi}^2) + \frac{m_0}{G}\sigma\,. \tag{2}$$

Using functional integral technique the quantized theory at finite temperature and density can be written in terms of the corresponding euclidean grand canonical partition function [16]:

$$Z = \mathrm{Tr}\,\exp\Big\{-\beta(H - \mu N)\Big\} = \frac{1}{Z_0}\int \mathcal{D}\Psi\mathcal{D}\Psi^\dagger\,\exp\Big\{\int_0^\beta \mathrm{d}\tau\int_V \mathrm{d}^3x(\mathcal{L} - \Psi^\dagger\mu\Psi)\Big\}\,, \tag{3}$$

where V is the volume of the system, β is the inverse temperature and μ is the chemical potential. The integration over the quarks can be done exactly, whereas for the integration over the mesons we use a large N_c saddle-point (mean-field) approximation. It means that the meson fields are treated classically – no meson loops are taken into account. Following refs. [16] we replace the integration over the imaginary time by a sum over fermionic Matsubara frequencies $k_0 \to (2n+1)\pi/\beta$. Finally we get for the effective action

$$\begin{aligned} S_{eff}(\mu,\beta) = -\ln Z = -\beta V N_c \sum_\alpha \left\{\frac{1}{2}(\epsilon_\alpha - \mu) + \frac{1}{\beta}\ln\Big[1 + \mathrm{e}^{-(\epsilon_\alpha-\mu)\beta}\Big]\right\} \\ + \beta\int_V \mathrm{d}^3x\Big[\frac{1}{2G}(\sigma^2 + \vec{\pi}^2) - \frac{m_0}{G}\sigma\Big]\,. \end{aligned} \tag{4}$$

The energies ϵ_α are eigenvalues of the one-particle hamiltonian h:

$$h\Phi_n \equiv \Big[\frac{\vec{\alpha}\cdot\vec{\nabla}}{i} + \gamma_0(\sigma + i\gamma_5\vec{\pi}\cdot\vec{\tau})\Big]\Phi_n = \epsilon_n\Phi_n \tag{5}$$

and Φ_n are eigenfunctions. The saddle-point solution makes the effective action stationary

$$\left.\frac{\partial S_{eff}}{\partial\sigma}\right|_{\sigma_c} = \left.\frac{\partial S_{eff}}{\partial\vec{\pi}}\right|_{\vec{\pi}_c} = 0\,, \tag{6}$$

with the number of particles N in the volume V

$$N = -\frac{1}{\beta}\left.\frac{\partial S_{eff}}{\partial\mu}\right|_{\sigma_c,\vec{\pi}_c}\,. \tag{7}$$

kept fixed. Here σ_c and $\vec{\pi}_c$ are the "classical" values of the meson fields and μ is the chemical potential related to the number of particles N.

The thermodynamical characteristics of a many-body system are specified by the thermodynamical potential

$$\Omega(\mu,\beta) \equiv \frac{S_{eff}(\mu,\beta)}{\beta V} \tag{8}$$

It should be noticed that the saddle-point solution $(\sigma_c, \vec{\pi}_c)$ minimizes not Ω but the Helmholtz free energy

$$F = \Omega - \mu\frac{\partial\Omega}{\partial\mu}\,, \tag{9}$$

with a constraint (7) and μ playing a role of a Lagrange multiplier.

In the mean-field approximation (leading order in $1/N_c$) the inverse meson propagator is given by the second variation of the effective action at the stationary point $\sigma_c, \vec{\pi}_c$:

$$K_\phi^{-1}(x-y) = \left.\frac{\partial^2 S_{eff}}{\partial\phi(x)\partial\phi(y)}\right|_{\phi_c}\,, \tag{10}$$

where ϕ stands for both meson fields. The on-shell meson masses correspond to the poles of the meson propagator

$$K_\phi^{-1}\left(q_0^2 = -m_\phi^2\right) = 0 \tag{11}$$

at $\vec{q} = 0$ and the physical quark meson coupling constants are given by the residue of the propagator at the pole

$$g_\phi^2 = \lim_{q^2\to -m_\phi^2}(q^2 + m_\phi^2)K_\phi(q^2)\,. \tag{12}$$

Due to the local four-fermion interaction the lagrangian (1) is not renormalizable and a regularization procedure with an appropriate cut-off Λ is needed to make the effective action finite. Actually only the part of the effective action $S_{eff}(\mu = 0, \beta = \infty)$, coming from the Dirac sea (negative-energy part of the spectrum), is divergent and we regularize it using the proper-time regularization

$$\mathrm{Tr}\ln\hat{A} \to -\mathrm{Tr}\int\limits_{\Lambda^{-2}}^{\infty}\frac{\mathrm{d}s}{s}\mathrm{e}^{-s\hat{A}}\,. \tag{13}$$

It is easy to check that the difference $S_{eff}(\mu,\beta) - S_{eff}(\mu = 0, \beta = \infty)$ is finite and does not need any regularization. Such a scheme is consistent with the usual regularization used in the solitonic calculations in the NJL model (see [19] and the references therein) in which only the divergent part of the effective action is regularized, whereas the finite valence part is not affected by the regularization. Moreover, any regularization of the medium part would suppress the temperature effects [9], since the positive part of the spectrum would be affected by the cutoff as well. Thus regularizing only the divergent Dirac sea part, we obtain the effective action in the form

$$\begin{aligned} S_{eff}(\mu,\beta) = \beta V N_c\Big\{\sum_{\epsilon_\alpha<0}\big[R^\Lambda_{3/2}(\epsilon_\alpha) + (\mu - \epsilon_\alpha)\big] + \frac{1}{\beta}\sum_\lambda \ln\Big[1 + \mathrm{e}^{-\beta(\epsilon_\alpha-\mu)}\Big]\Big\} \\ + \beta\int_V \mathrm{d}^3x\left[\frac{1}{2G}\left(\sigma^2 + \vec{\pi}^2\right) - \frac{m_0}{G}\sigma\right] \end{aligned} \tag{14}$$

where the proper-time regularization function is given by

$$R^\Lambda_\alpha(\epsilon) = \frac{1}{\sqrt{4\pi}}\int_{\Lambda^{-2}}^{\infty}\frac{d\tau}{\tau^\alpha}\mathrm{e}^{-\tau\epsilon^2}\,. \tag{15}$$

Since the Dirac sea does not contribute to the baryon number, expression (7) is finite and does not include any regularization.

III. FIXING THE MODEL PARAMETERS

In the vacuum ($\mu = 0, T = 0$) the stationary conditions (6) lead to a translationary invariant solution

$$\sigma_c = M_0 \qquad \text{and} \qquad \vec{\pi}_c = 0\,, \tag{16}$$

which breaks the chiral symmetry. Due to the non-zero vacuum expectation value σ_c the quarks acquire a constituent mass M_0. The latter is a solution of the gap equation (stationary condition for the sigma field) which in the case of proper-time regularization has the form

$$M_0 = \, G\, M_0 \frac{N_c}{2\pi^2} \int_{\Lambda^{-2}}^{\infty} \frac{\mathrm{d}s}{s^2} \mathrm{e}^{-sM^2} + m_0 \equiv m_0 - G < \bar{\Psi}\Psi > \tag{17}$$

with $< \bar{\Psi}\Psi >$ being the quark condensate. Using (10) it is straightforward (see ref. [17]) to evaluate the meson propagator:

$$K_\phi(q^2) = \frac{1}{Z_p(q^2)} \frac{1}{q^2 + \delta_{\phi\sigma} 4M_0^2 + \frac{m_0}{GM_0 Z_p(q^2)}}\,. \tag{18}$$

The function $Z_p(Q^2)$ corresponds to a quark loop with two pseudoscalar-isovector insertions ($i\gamma_5\tau_a$). Here we present only the proper-time regularized expression for it:

$$Z_p(Q^2) = \frac{N_C}{4\pi^2} \int\limits_0^1 \mathrm{d}u \int\limits_{\Lambda^{-2}}^{\infty} \frac{\mathrm{d}s}{s} \mathrm{e}^{-s\left[M_0^2 + \frac{q^2}{4}(1-u^2)\right]} = \frac{N_C}{4\pi^2} \int\limits_0^1 \mathrm{d}u \Gamma\Big(0, \frac{M_0^2 + \frac{q^2}{4}(1-u^2)}{\Lambda^{-2}}\Big)\,, \tag{19}$$

with $\Gamma(0, x)$ being the incomplete gamma function.

We fix the parameters of the model, namely the current mass m_0, the cutoff Λ and the coupling constant G in the vacuum sector reproducing the physical pion mass $m_\pi = 140$ MeV and the pion decay constant $f_\pi = 93$ MeV. In fact, it leads to the Goldberger-Treiman (GT) relation on the quark level

$$M_0 = g_\pi f_\pi\,, \tag{20}$$

and one also recovers the Gell-Mann - Oakes - Renner (GMOR) relation:

$$m_\pi^2 f_\pi^2 = -m_0 < \bar{\Psi}\Psi > + O(m_0^2)\,. \tag{21}$$

As usual, the last model parameter, the coupling constant G, is eliminated in favor of the constituent mass M_0 using the gap equation (17). In fact, a value around $M_0 = 420$ MeV is needed to describe properly the nucleon properties [19] and we will use this value in our computations. The corresponding value of the proper-time cutoff is $\Lambda = 640$ MeV.

IV. PHASE TRANSITION AND MESON PROPERTIES AT FINITE TEMPERATURE AND DENSITY

In this section we investigate the translationary invariant medium solution at finite temperature T and density ρ. For the model parameters m_0, Λ and M_0 we use the values fixed in the vacuum.

A. Quark medium

In this subsection we consider the medium as a Fermi sea of quarks. Solving (6) together with the number of particles N in the volume V (7) kept fixed, we get the the medium solution in the form

$$\sigma_c = M \qquad \text{and} \qquad \vec{\pi}_c = 0\,. \tag{22}$$

As in the vacuum case, only the "classical" value of the scalar field can be non-zero and we call it the constituent quark mass M in medium. This mass M is a solution of the equation of motion the scalar field

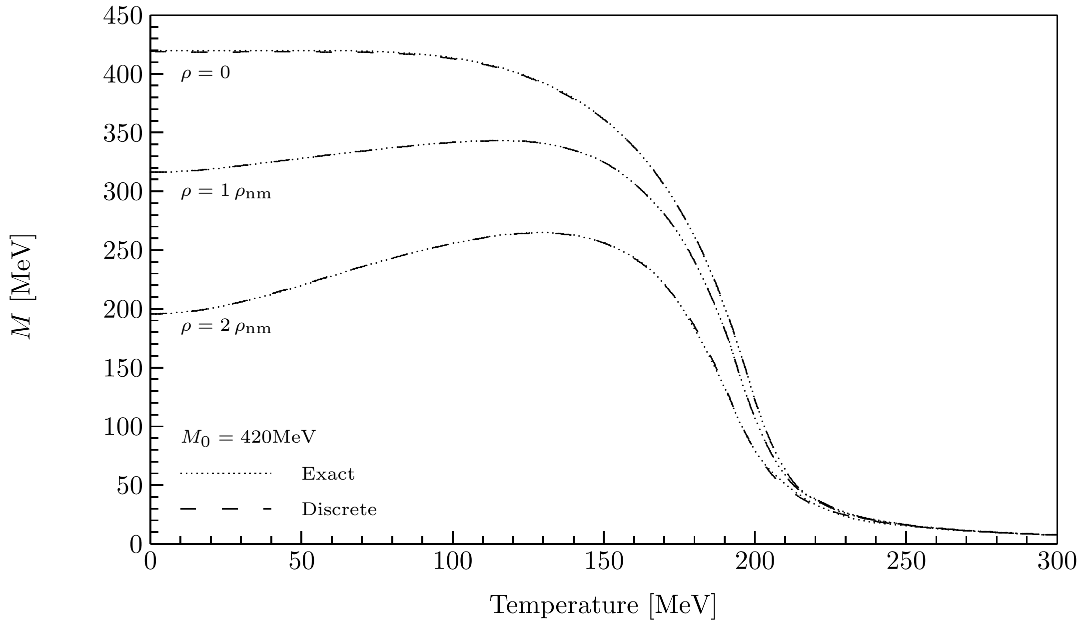

FIG. 1. Constituent quark mass M as a function of temperature for different densities. At vanishing both temperature and density it is fixed to $M_0 = 420$ MeV. The results with plane-wave basis (dotted lines) are compared to those of the finite-discrete basis (dashed lines).

$$M = -\,G\,N_c\left[\sum_{\epsilon_n<0}\bar{\Phi}_n\Phi_n R^{\Lambda}_{1/2}(\epsilon_n)\epsilon_n + \sum_{\epsilon_n>0}\frac{\bar{\Phi}_n\Phi_n}{1+\mathrm{e}^{(\epsilon_n-\mu)\beta}} - \sum_{\epsilon_n<0}\frac{\bar{\Phi}_n\Phi_n}{1+\mathrm{e}^{-(\epsilon_n-\mu)\beta}}\right] + m_0\,, \tag{23}$$

with the proper-time regulator $R^{\Lambda}_{1/2}(\epsilon_n)$ in the form (15). The thermodynamical potential Ω is given by (8):

$$\begin{aligned}\Omega(\mu,\beta) = N_c\sum_{\epsilon_n<0}\Big[R^{\Lambda}_{3/2}(\epsilon_n) - R^{\Lambda}_{3/2}(\epsilon^0_n) + (\mu-\epsilon_n)\Big] - \frac{N_c}{\beta}\sum_n \ln\Big[1+\mathrm{e}^{-\beta(\epsilon_n-\mu)}\Big]\Big\}\\ + \frac{1}{V}\int_V \mathrm{d}^3x\left[\frac{1}{2G}(M^2-M_0^2) - \frac{m_0}{G}(M-M_0)\right].\end{aligned} \tag{24}$$

Here, we subtract the vacuum value of the thermodynamical potential $\Omega(0,0)$. The energies ϵ^0_n are the eigenvalues of the hamiltonian h for the vacuum meson configuration (16). Using (7) the baryon density ρ_B can be written as

$$\rho_B = -\frac{1}{N_c}\frac{\partial\Omega}{\partial\mu} = \sum_{\epsilon_n>0}\frac{1}{1+\mathrm{e}^{(\epsilon_n-\mu)\beta}} - \sum_{\epsilon_n<0}\frac{1}{1+\mathrm{e}^{-(\epsilon_n-\mu)\beta}}\,, \tag{25}$$

and the free energy (9) is given by

$$F(\mu,\beta) = \Omega(\mu,\beta) + N_c\mu\rho_B\,. \tag{26}$$

In the calculations we use a plane wave basis as well as a quasi-discrete basis and numerical method of Ripka and Kahana [20] for solving the eigenvalue problem (5) by putting the system in a spherical box of a large radius D. The basis is made discrete by imposing a boundary condition at D. It is also made finite by introducing a numerical cut-off $k_{\max}$ for the momenta of the basis states. Obviously both parameters are technical and the results do not depend on their particular values. The typical values which we use, are $D \approx 18/M_0$ and $k_{\max} \approx 8M_0$. The volume V is taken to be a bit smaller than the box in order to avoid finite-size-box effects and the baryon number is given by

$$N_B = \rho_B V\,. \tag{27}$$

The results for the constituent mass M as a function of temperature for different densities are presented in fig. 1 for both the plane wave and the quasi-discrete basis. As can be seen the quasi-discrete basis provides a quite good description of the continuum.

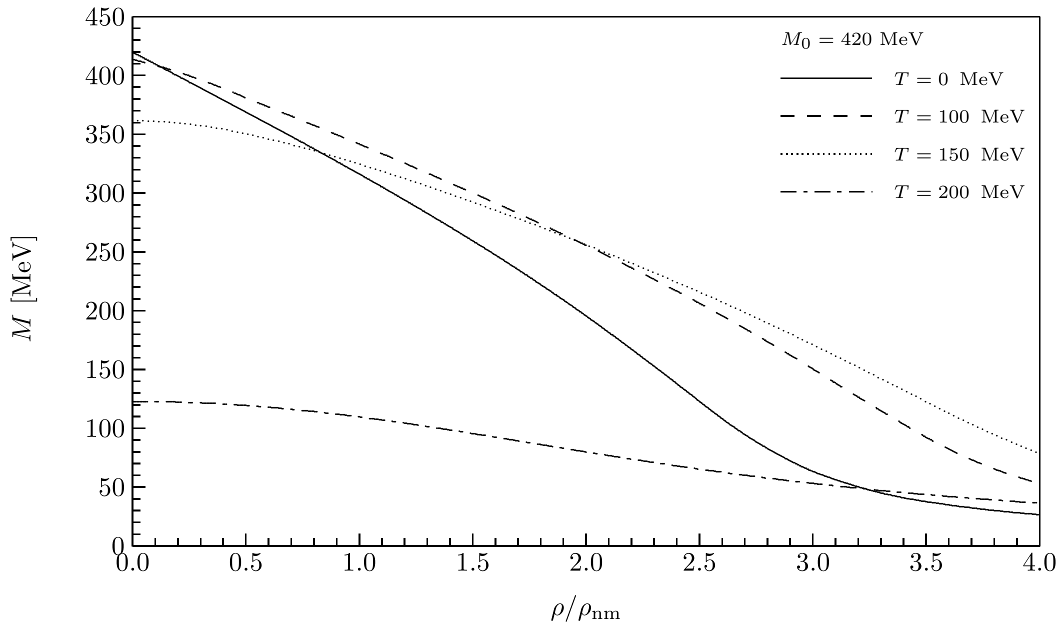

FIG. 2. Constituent quark mass M as a function of density for different temperatures.

At finite density the constituent mass (or the chiral oder parameter $< \bar{\Psi}\Psi >$) is a non-monotonic function of temperature. It means that in a hot matter M is less affected than in the case of cold matter. This also can be seen on fig. 2 where M is presented as function of density for different temperatures. At some critical values of temperature and/or density the constituent mass M is reduced to the current mass m_0 (the quark condensate $< \bar{\Psi}\Psi >$ vanishes) which is an indication for a a transition to the Wigner phase where the chiral symmetry is not spontaneously broken. The corresponding equation of state (pressure versus density) can be found in ref. [9]. The critical $\rho - T$ chiral phase transition diagram is shown in fig. 3. At lower temperature values $T < 90$ MeV we have a clear first order transition which change to a second order one at higher temperatures. The critical value of the temperature is close to 200 MeV and for the density is about 2.5 ρ_{nm} (nuclear matter density $\rho_{nm} = 0.16$ fm^{-3}).

The meson masses in medium are defined as the lowest zero solution of the inverse meson propagators

$$K_\phi^{-1}(q_0^2 = -m_\phi^2) = 0\,, \tag{28}$$

at $\vec{q} = 0$. The latter is given by the second variation of the effective action with respect to the meson fields

$$\begin{aligned} K_\phi^{-1}(q_0^2) = \Big(\frac{q_0^2}{4} + \delta_{\phi\sigma} M^2\Big)\Bigg\{ Z_p(q_0^2) - 4N_c\mathcal{P}\int \frac{\mathrm{d}^3k}{(2\pi)^3}\frac{1}{\epsilon_k^2 + \frac{q_0^2}{4}}\left[\frac{1}{1+\mathrm{e}^{\beta(\epsilon_k-\mu)}} + \frac{1}{1+\mathrm{e}^{-\beta(\epsilon_k+\mu)}}\right]\Bigg\} \\ + \frac{m_0}{GM}\,. \end{aligned} \tag{29}$$

In the derivation of (29) we use the plane wave basis and eq.(22). $\mathcal{P}$ means principle value. From (29) it is clear that in the Goldstone phase the scalar and pseudoscalar mesons differ in mass. In the chiral limit the pions remain massless Goldstone bosons also in the medium, whereas the sigma mass follows the constituent quark mass. Both GMOR and GT relations are also valid in medium. Since in the Wigner phase the constituent mass M vanishes up to the current mass m_0, the mesons become degenerated in mass and appear as parity-doubled mesons. In fact, in the Wigner phase due to the Fermi sea contribution, even in the chiral limit $m_0 = 0$ the mesons have a non-zero mass, which increases with temperature (density). It is illustrated in fig. 4 where our results for the meson masses in medium are shown as a function of temperature at three different densities. Similar results are obtained by Zhuang et al. [12] in the case of finite temperature and vanishing chemical potential.

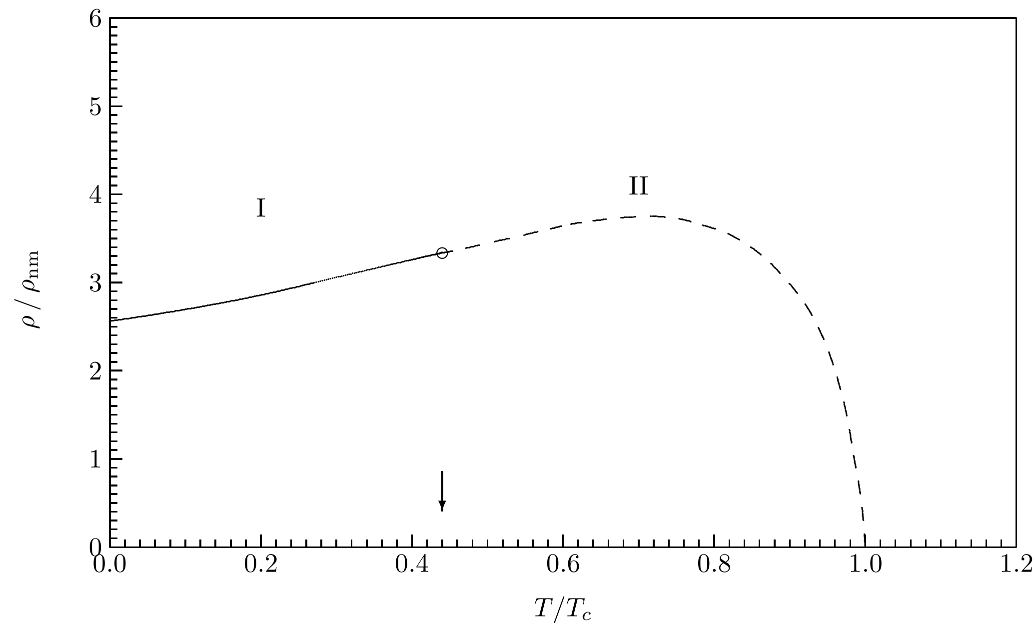

FIG. 3. $\rho - T$ critical phase diagram. The lines separates the Goldstone phase with the chiral symmetry breaking from the Wigner phase where the chiral symmetry is restored. The solid line shows the critical values for which there is a first order transition, whereas the dashed line corresponds to the second order one. The arrow shows the temperature at which the order is changed.

Since the NJL model lacks confinement there is a coupling to the $\bar{q}q$ continuum. In the Goldstone phase this coupling is unphysical but it does not affect much the structure of the scalar and pseudoscalar mesons – for the pion we have a δ-peak far from a very weak $\bar{q}q$ continuum and for the sigma meson there is a very sharp peak at the $\bar{q}q$ threshold ($4M^2$). The reason is that the pions play the role of Goldstone bosons and the sigma mesons are their chiral partner. In the Wigner phase, however, after the deconfinement transition (suggested by the lattice results) the $\bar{q}q$ channel is physically open. The meson states are not more stable.

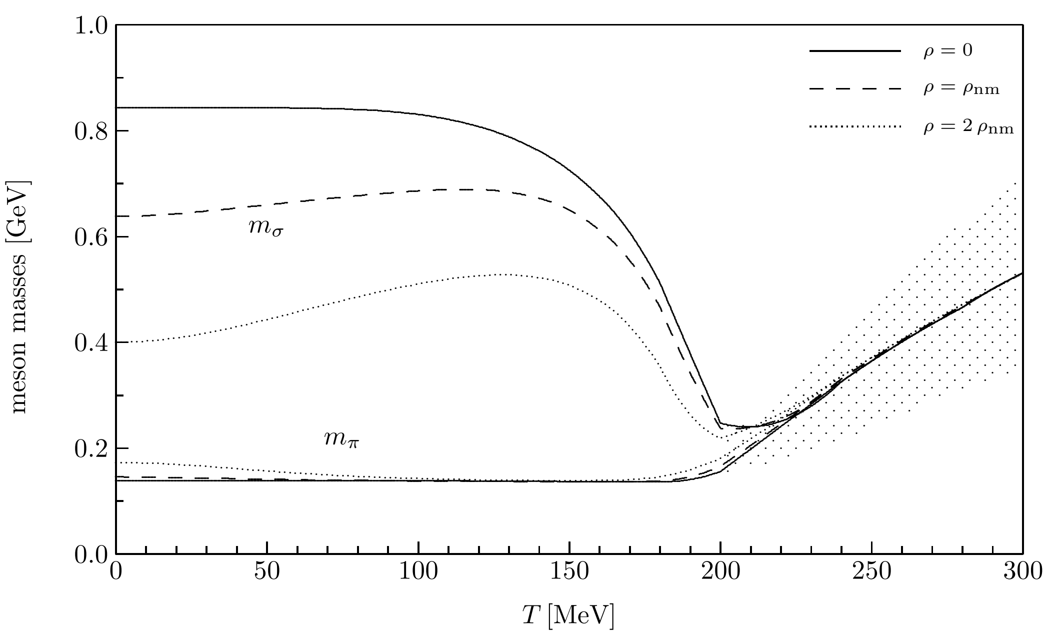

FIG. 4. Meson masses in the quark medium as a function of temperature for different densities. The shadowed area shows the width of the meson resonances.

They appear as resonances in the $\bar{q}q$ continuum and the poles of the meson propagators are complex. In this case one can identify the meson resonances with the peaks in the spectral density (imaginary part of the meson propagator). The spectral density is shown on fig. 5 for some different temperatures above the critical one. As can be seen, there is indeed a clear resonance structure. Close to the critical $T_{cr} \approx 200$ MeV the peak is rather narrow. With the increasing temperature the resonance structures become weaker and at around 300 MeV is almost washed out. The position of the peak moves to higher energies and its width increases as well which implies that the mesons in the Wigner phase are rather soft modes than elementary excitations [5]. It is also interesting to notice that in this phase the meson masses show almost no density dependence. To conclude, according to the above model picture, at higher temperatures one might expect a smooth dehadronization transition. In this range, not the hadron but the quark and gluon degrees of freedom are expected to be relevant. Apparently, the missing explicit gluonic degrees of freedom in the NJL model limits its applicability at high temperatures.

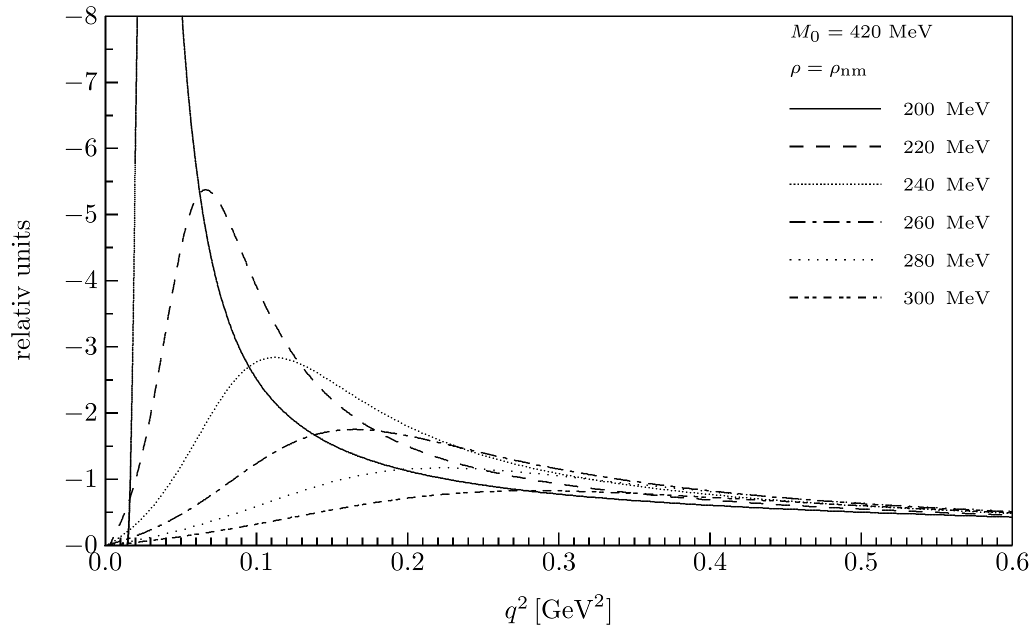

FIG. 5. Imaginary part of the pion propagator (spectral density) for nuclear matter density and different temperatures.

B. Medium of nucleons

In the previous subsection we have considered the quark medium. In this case at finite temperature the single quarks from the Dirac sea are allowed be excited and occupy the levels in positive part of the spectrum (Fermi sea) leaving holes (antiquarks) in the Dirac sea. Apparently, because of confinement, at T and ρ below the critical values for the chiral and deconfinement transitions it is forbidden. As an alternative to this picture in this subsection we use a hybrid model in which instead of quarks we consider a Fermi sea of nucleons. The Dirac sea consists of quarks and it determines the vacuum sector. In this model the mesons are still $\bar{q}q$ excitations but they are also directly coupled to the nucleons of the Fermi sea. Such a model have been proposed and used by Jaminon et al. [17] to study the the chiral symmetry restoration in a cold nucleon medium. At finite temperature only nucleons (colorless N_c quark clusters) can be removed from the Dirac sea and occupy the levels in the Fermi sea. In this model picture, removing nucleons from the Dirac to the Fermi sea, one creates antinucleons. Apparently, it is physically reasonable to apply this hybrid model only for temperature and density below their critical values. As we see later, almost simultaneously with the chiral phase transition there is a delocalization of the soliton in the model, which means that after the phase transition it is consistent to consider a quark medium rather than the nucleon one.

Since in the present scheme the contribution of the Dirac sea is separated from the one of the Fermi sea, it is straightforward to write down the medium contribution to the thermodynamical potential (effective action) in terms of nucleons:

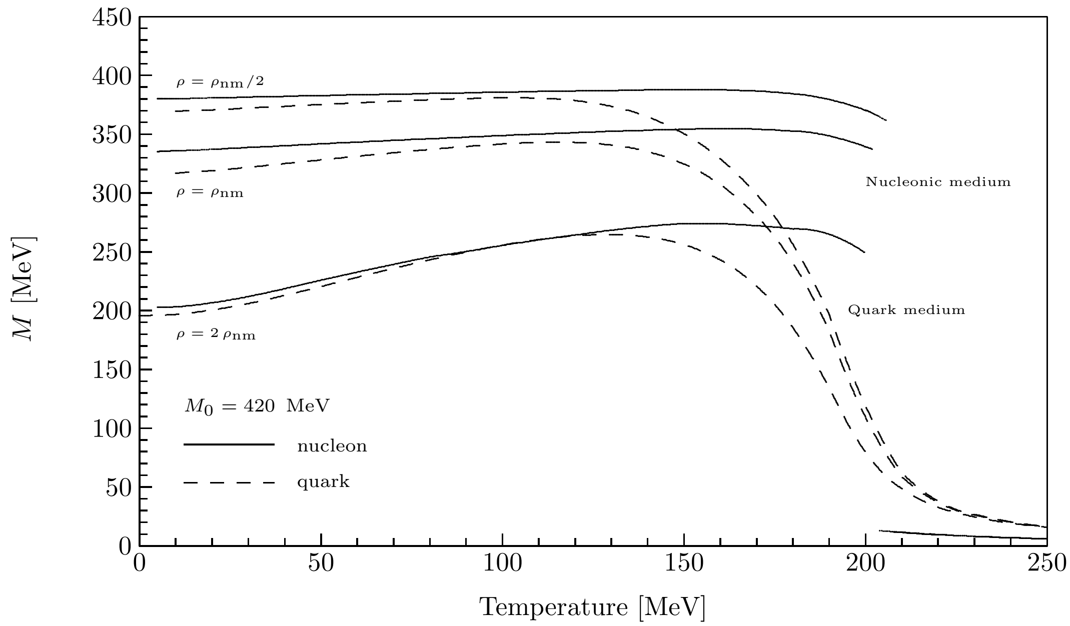

FIG. 6. Constituent quark mass M as a function of temperature for different densities in the quark (dashed lines) and nucleon medium (solid lines).

$$\Omega^N_{med} = \sum_{\epsilon^N_n<0} (\mu^N - \epsilon^N_n) - \frac{1}{\beta}\sum_n \ln\Big[1 + \mathrm{e}^{-\beta(\epsilon^N_n - \mu^N)}\Big] \,. \tag{30}$$

The energies ϵ^N_n are the solutions of the Dirac equation

$$h_N \Phi^N_n \equiv \Big[\frac{\vec{\alpha}\cdot\vec{\nabla}}{i} + \beta g_N(\sigma + i\gamma_5 \vec{\pi}\cdot\vec{\tau})\Big]\Phi^N_n = \epsilon^N_n \Phi^N_n \,. \tag{31}$$

The meson fields are coupled to the nucleons with a coupling constant g_N which relates the nucleon mass to the nonzero expectation value of the scalar meson field (constituent quarks mass M_0) in vacuum

$$M_N = g_N M_0 \,. \tag{32}$$

As in the quark spectrum, there is a gap of $2M_N$ in the nucleon spectrum which separates the negative part of the spectrum from the positive one. In the nucleon medium, we also assume that g_N remains unchanged which means that a relation similar to (32) is valid also in medium. Since we are able to calculate the mass of the nucleon as a soliton, later we will check this approximation. The chemical potential μ^N is fixed by the baryon density

$$\rho_B = \sum_{\epsilon^N_n>0} \frac{1}{1 + \mathrm{e}^{(\epsilon^N_n - \mu^N)\beta}} - \sum_{\epsilon_n<0} \frac{1}{1 + \mathrm{e}^{-(\epsilon^N_n - \mu^N)\beta}} \,, \tag{33}$$

(or equivalently by the baryon number (27)). It is interesting to make a parallel with the approach of Walecka et al. [25]. Both models contain nucleons interacting locally with meson fields. However, in contrast to the Walecka model, where the meson fields are fundamental and the meson properties are introduced as phenomenological parameters, in the hybrid model the meson fields appears basically as $\bar{q}q$ pairs. Hence, we are able to calculate the meson masses and the physical coupling constants for a given constituent mass in vacuum as well as in medium. In the latter case, there is a change of the chiral condensate which leads to a modification of the meson properties as well. Moreover, within the present model the nucleon can be described as a soliton, which allows for a check of the basic assumption of the model, namely of the universality of the local coupling of the nucleons to the meson fields (32).

As in the case of a quark medium, only the scalar field can have a non-zero expectation value in nucleon medium (constituent quark mass in medium) which is a solution of the corresponding equation of motion (23) with a finite-temperature part written now in terms of nucleons

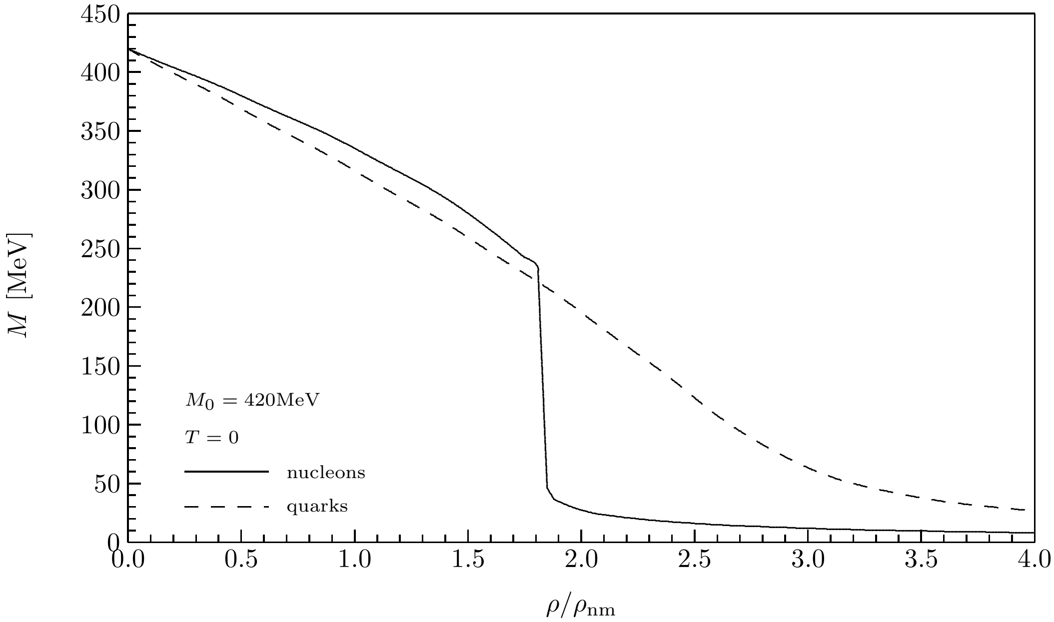

FIG. 7. Constituent quark mass M as a function of density at $T = 0$ in the quark (dashed line) and nucleon medium (solid line).

$$N_c \sum_{\epsilon_n>0} \frac{\bar{\Phi}_n \Phi_n}{1+\mathrm{e}^{(\epsilon_n-\mu)\beta}} - N_c \sum_{\epsilon_n<0} \frac{\bar{\Phi}_n \Phi_n}{1+\mathrm{e}^{-(\epsilon_n-\mu)\beta}} \longrightarrow \sum_{\epsilon_n^N>0} \frac{\bar{\Phi}_n^N \Phi_n^N}{1+\mathrm{e}^{(\epsilon_n^N-\mu^N)\beta}} - \sum_{\epsilon_n^N<0} \frac{\bar{\Phi}_n^N \Phi_n^N}{1+\mathrm{e}^{-(\epsilon_n^N-\mu^N)\beta}} \tag{34}$$

together with the constraint $\rho_B = const.$

The results for M are presented in fig.6 at finite density as a function of T and on fig.7 as a function of density at $T = 0$. At low T values the curves are close to those of the quark matter which means that the use of a quark matter instead of nucleon one is a reasonable approximation which is not true for higher temperatures. Already at intermediate temperatures $T > 120$ MeV they start to deviate significantly. Whereas the quark matter is quite soft against thermal fluctuations, the nucleon matter is much stiffer and the corresponding chiral phase transition is rather sharp. Both the chiral condensate and the constituent quark mass show a discontinuity at the critical temperature. In fact, the system jumps between two minima. The later is suggests that we have to deal with a first order phase transition even in the case of vanishing density in contrast to the case of quark matter where at temperatures $T > 80$ MeV it is a second order one. The critical temperature is slightly larger than in the quark case and as we will see later it almost coincides with the critical temperature for the delocalization of the soliton which makes this model picture consistent. The results at $T = 0$ and finite ρ suggest a different model scenario. The critical density ($\approx$ two times the nuclear matter density) for the chiral transition, which in fact, as we will see later, is also the critical density for the delocalization of the soliton in cold nucleon matter, is smaller than the one for the quarks matter. This model scenario suggests a delocalization transition from cold nucleon matter to quark one before the chiral phase transition to take place.

According to the present model picture, at some critical temperature one expects a chiral phase transition together with a delocalization transition from nucleon matter to quark one. Apparently, it would lead to drastic structural changes in the system. It can be clearly seen from the corresponding EOS (pressure versus baryon density) for different temperatures plotted in fig.8. On this figure we combine the results from the nucleon matter (below the critical temperatures) in the hybrid model with those of the quark matter after the transition. Since we do not include vector mesons in the hybrid model, we are not able to reproduce the nuclear matter saturation at zero temperature and finite density which in the Walecka approach is due to the interplay between the σ-meson attraction and the ω-meson repulsion. All curves in fig.8 show rapid change discontinuity at the delocalization transitions from nucleon to quark matter. Similar behavior can be found on the fig. 9 and fig. 10 for the pressure and energy density as a function of T for different densities.

Apart from the medium part written in terms of nucleons (34) the inverse meson propagators in the nucleon medium have the same structure (29) as in the quark medium. In chiral limit the pions are massless Goldstone bosons whereas the sigma mass is given by $2M$. Like the constituent mass M the mass of the sigma meson in the nucleon medium has a discontinuity at the critical temperature.

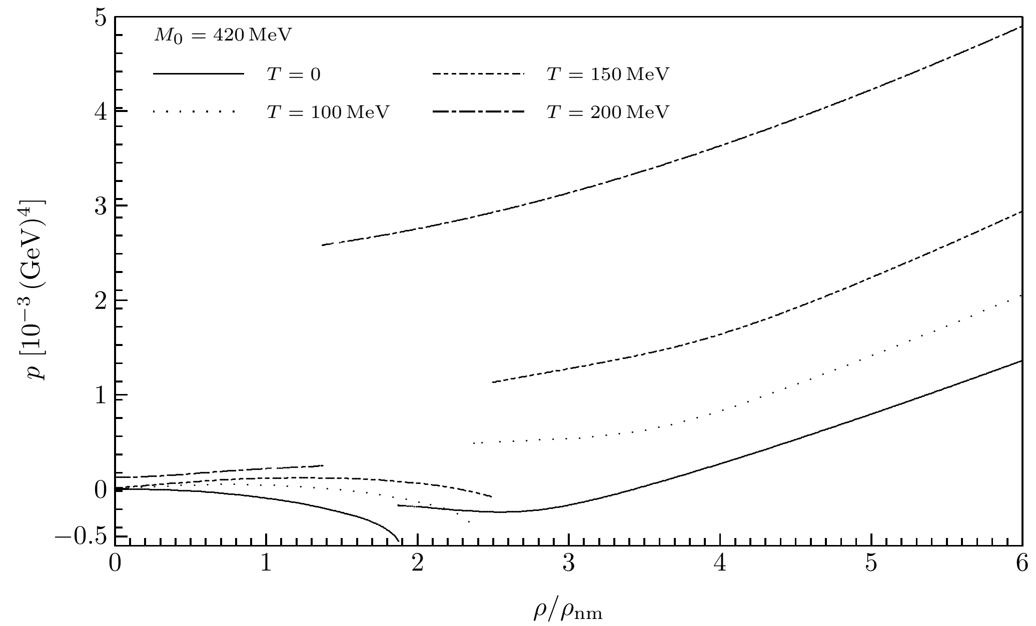

FIG. 8. EOS (pressure versus density) for different temperatures. The pressure of the vacuum is subtracted. The discontinuities correspond to a phase transition from a nucleon to a quark matter.

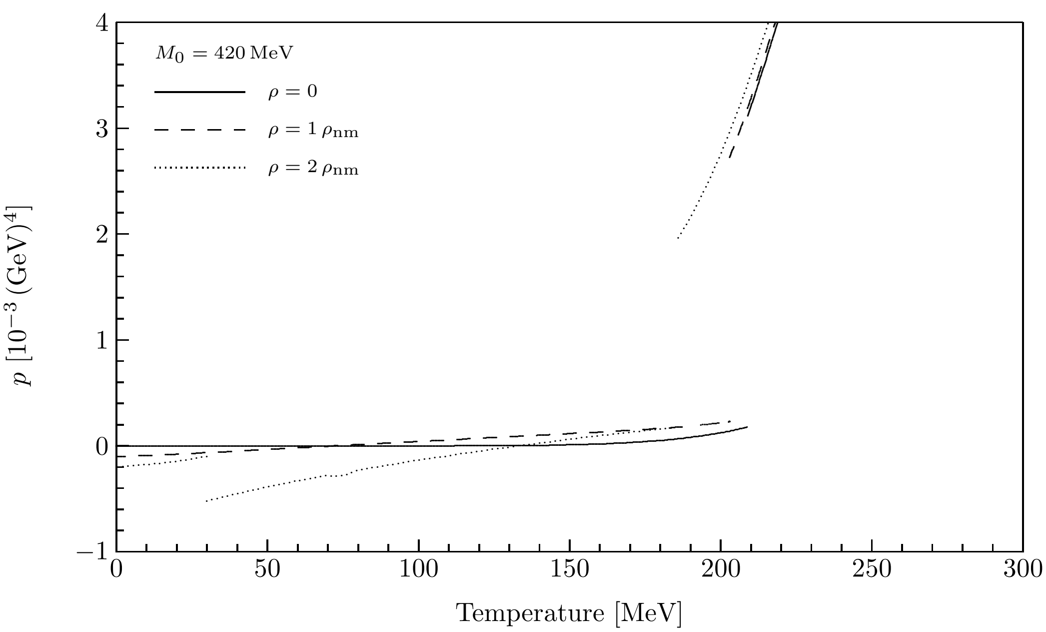

FIG. 9. Pressure as a function of temperature for different densities. The discontinuities show the phase transition from the nucleon to quark matter.

Our considerations are done in the mean-field approximation and suffer from the fact that meson quantum (loop) effects are not taken into account. However, as it is estimated in [11,12] the meson quantum effects may play a dominant role at low temperatures and vanishing density, whereas close to the critical temperature their contribution to the bulk thermodynamical variables is of order of 10 % or even less. Hence, following the result of [11,12], one expects that the meson fluctuations would not change the model scenario for the phase transition presented above.

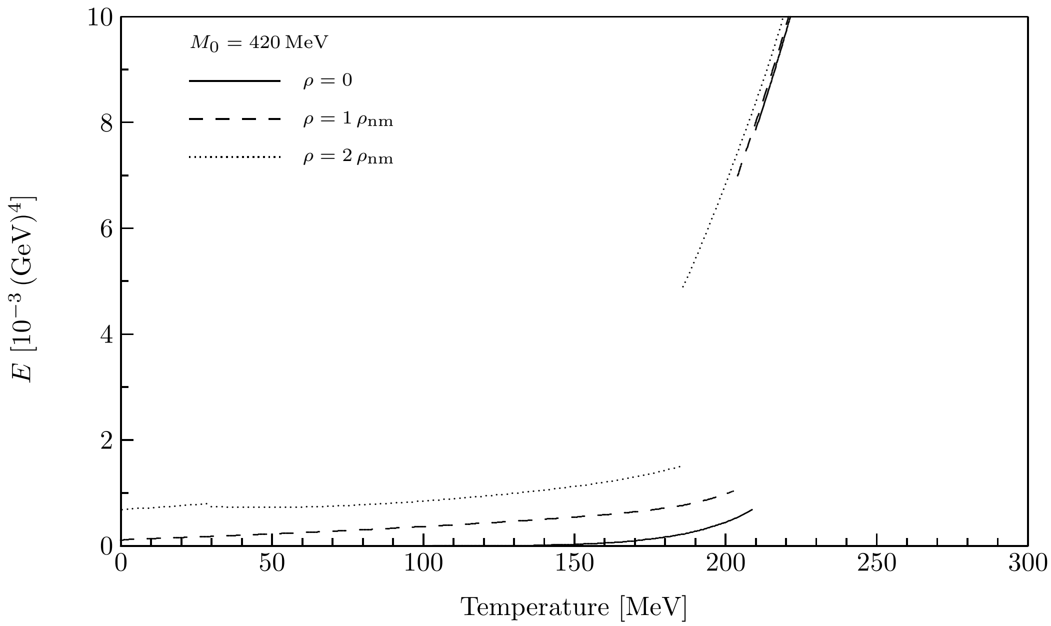

FIG. 10. Energy density as a function of temperature for different densities. The discontinuities show the phase transition from the nucleon to quark matter.

V. NUCLEON AS A SOLITON IN MEDIUM

As a next step we solve the $B = 1$ soliton sector in a hot medium. We look for a localized bound solution (soliton) of N_c valence quarks interacting with the Fermi and Dirac sea both polarized due to the interaction.

A. In quark medium

First we consider the case of Fermi and Dirac sea both filled with quarks of a constituent mass M. The thermodynamical potential (effective action) includes an explicit valence quark contribution:

$$\Omega(\mu,\beta) = N_c\theta(\epsilon_{val})\epsilon_{val} + N_c \sum_{\epsilon_n<0}\Big[R^{\Lambda}_{3/2}(\epsilon_n) - R^{\Lambda}_{3/2}(\epsilon^0_n) + (\mu-\epsilon_n)\Big] - \frac{N_c}{\beta}\sum_{\epsilon_n\neq val}\ln\Big[1+\mathrm{e}^{-\beta(\epsilon_n-\mu)}\Big]\Big\}$$
$$+\frac{1}{V}\int_V \mathrm{d}^3x\left[\frac{1}{2G}(M^2-M_0^2) - \frac{m_0}{G}(\sigma - M_0)\right]. \tag{35}$$

since the valence level is always occupied by N_c quarks.

The meson fields are assumed to be in a hedgehog form

$$\sigma(\mathbf{r}) = \sigma(r) \qquad \text{and} \qquad \vec{\pi}(\mathbf{r}) = \hat{\mathbf{r}}\pi(r) \tag{36}$$

and are restricted on the chiral circle

$$\sigma^2 + \vec{\pi}^2 = M^2\,. \tag{37}$$

The latter is an *ad hoc* nonlinear constraint which reflects the fact that the pions as almost massless Goldstone bosons play a privileged role of dynamical mesons in this low-energy region [21].

From (6) one gets the equations of motion for the meson fields

$$\sigma(\vec{r}) = -G\,N_c\left\{\sum_n \bar{\Phi}_n(\vec{r})\Phi_n(\vec{r})R^{\Lambda}_{1/2}(\epsilon_n)\epsilon_n + \theta(\epsilon_{val})\bar{\Phi}_{val}(\vec{r})\Phi_{val}(\vec{r}) + \sum_{\epsilon_n>0}\frac{\bar{\Phi}_n(\vec{r})\Phi_n(\vec{r})}{1+\mathrm{e}^{(\epsilon_n-\mu)\beta}}\right.$$
$$\left. - \sum_{\epsilon_n<0}\frac{\bar{\Phi}_n(\vec{r})\Phi_n(\vec{r})}{1+\mathrm{e}^{-(\epsilon_n-\mu)\beta}}\right\} + m_0 \tag{38}$$

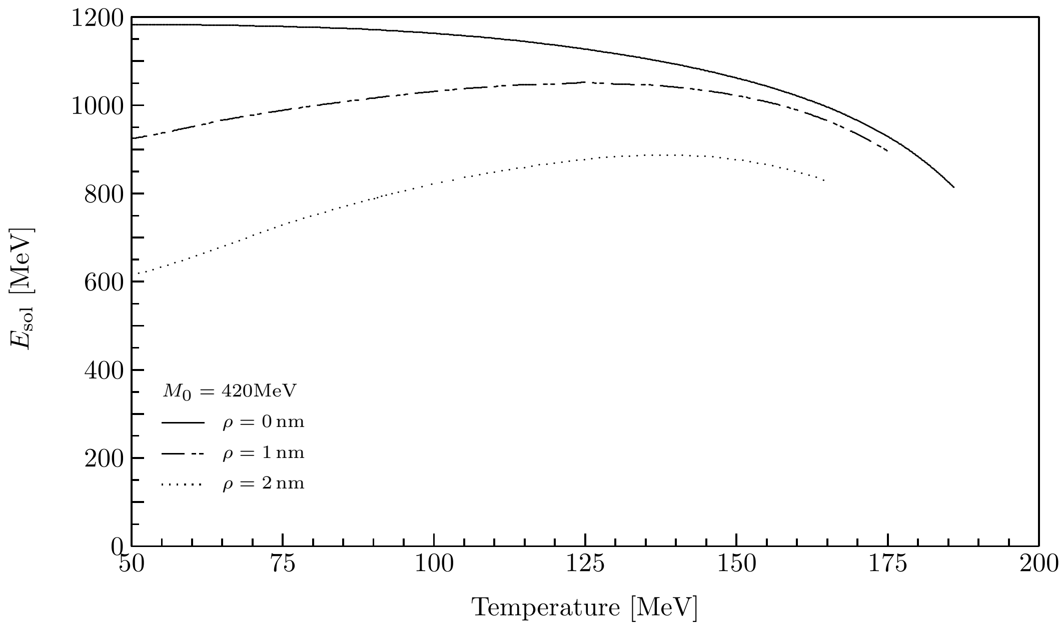

FIG. 11. Soliton energy as a function of the temperature at three different quark matter densities.

and

$$\pi(\vec{r}) = -G\,N_c\left\{\sum_n \bar{\Phi}_n(\vec{r})i\gamma_5(\hat{\mathbf{r}}\cdot\vec{\tau})\Phi_n(\vec{r})R^{\Lambda}_{1/2}(\epsilon_n) + \theta(\epsilon_{val})\bar{\Phi}_{val}(\vec{r})i\gamma_5(\hat{\mathbf{r}}\cdot\vec{\tau})\Phi_{val}(\vec{r})\right.$$
$$\left. + \sum_{\epsilon_n>0}\frac{\bar{\Phi}_n(\vec{r})i\gamma_5(\hat{\mathbf{r}}\cdot\vec{\tau})\Phi_n(\vec{r})}{1+\mathrm{e}^{(\epsilon_n-\mu)\beta}} - \sum_{\epsilon_n<0}\frac{\bar{\Phi}_n(\vec{r})i\gamma_5(\hat{\mathbf{r}}\cdot\vec{\tau})\Phi_n(\vec{r})}{1+\mathrm{e}^{-(\epsilon_n-\mu)\beta}}\right\}. \tag{39}$$

We use a numerical self-consistent iterative procedure based on a method proposed by Ripka and Kahana [20]. The procedure consists in solving in an iterative way the Dirac equation (5) together with the equations of motion of the meson fields - eqs.(38),(39), and the constraint $N_B = const$ (27) which is actually a condition fixing the chemical potential μ.

The techniques for the numerical procedure are well known [22,23] for the case of vanishing chemical potential where there is no contribution coming from the positive continuum. They can be easily adopted to the case of finite chemical potential and temperature where an additional source (Fermi sea quarks) for the meson fields appears in eqs.(38),(39).

In the case of fixed T and ρ the proper way to describe the equilibrium state of a thermodynamical system is to use the free energy (9). Hence, the energy of the $B = 1$ soliton (effective soliton mass) is given by the change of the free energy when the N_c valence quarks are added to the medium. Apparently, in this the baryon number of the system increases by one, $N_B + 1$. Since due to the presence of the soliton both the Fermi sea and the Dirac sea are getting polarized, subtracting the free energy $F(\mu_0, \beta)$ of the unperturbed Fermi and Dirac sea (translationally invariant medium solution), the soliton energy is given by the sum of the energy of the valence quarks and the contributions due to the polarization of both continua:

$$E_{sol} \;=\; N_c\eta_{val}\epsilon_{val} + N_c\sum_{\epsilon_n<0}\Big[R^{\Lambda}_{3/2}(\epsilon_n) + (\mu-\epsilon_n)\Big] - \frac{N_c}{\beta}\sum_{\epsilon_n\neq val}\ln\Big[1+\mathrm{e}^{-\beta(\epsilon_n-\mu)}\Big]$$

$$- \frac{1}{V}\int_V \mathrm{d}^3x \frac{m_0}{G}\sigma + \mu N_c \rho_B - F(\mu_0, \beta)\,. \tag{40}$$

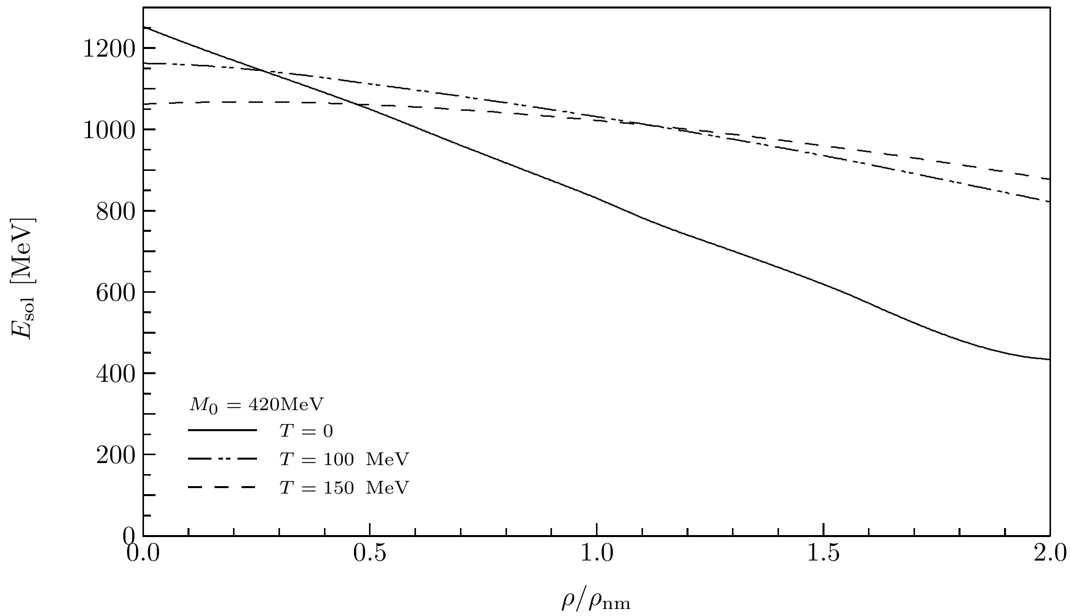

FIG. 12. Soliton energy as a function of density for vanishing as well as finite temperature values.

The chemical potential μ_0 fixed by the constrain $N_B = const$ (27) corresponds to the translationally invariant medium solution. The soliton baryon density distribution is split in valence, sea and medium parts:

$$\rho_{sol}(\vec{r}) = \theta(\epsilon_{val})\Phi^\dagger_{val}(\vec{r})\Phi_{val}(\vec{r}) + \frac{1}{2}\sum_{\epsilon_n}\Phi^\dagger_n(\vec{r})\Phi_n(\vec{r})\mathrm{sgn}(-\epsilon_n)$$
$$+ \sum_{\epsilon_n > M}\frac{\Phi^\dagger_n(\vec{r})\Phi_n(\vec{r})}{1+\mathrm{e}^{(\epsilon_n-\mu)\beta}} - \sum_{\epsilon_n<0}\frac{\Phi^\dagger_n\Phi_n}{1+\mathrm{e}^{-(\epsilon_n-\mu)\beta}} - \rho_B\,. \tag{41}$$

Because of the constraint $N_B = const$, only the first term in r.h.s. of (41) contributes to the baryon number $B = 1$. The soliton m.s.radius defined as

$$< r^2 >_{sol} = \int \mathrm{d}^3 r r^2 \rho_{B=1}(\vec{r}) \tag{42}$$

can be used to measure the spatial extension of the soliton.

Our results for the soliton energy as a function of the temperature for vanishing as well as for finite density values are presented in fig. 11. For completeness on the fig. 12 we also present the soliton energy as a function of density at vanishing [14] as well as at finite temperature.

We do not find a localized solution (soliton) at temperatures larger than a critical value $T^B_{cr} \approx 180$ MeV. It means that at large enough values the temperature effects simply disorder the system and destroys the soliton. Within the present model picture this may be interpreted as an indication for a delocalization of the nucleon in hot medium but, however, one should keep in mind that the model lacks confinement. It should be mentioned that a similar effect has been found in the Skyrme model [24].

As can be seen from fig. 11 the temperature effects for the soliton are much weaker than the finite density effects. In the case of finite T and zero ρ the soliton energy shows almost no reduction for temperatures not close to the critical one, whereas in the opposite case of finite ρ and zero T the soliton energy is linearly decreasing with the density. In the hot medium (both T and ρ finite) the temperature clearly suppresses the finite density effects and stabilizes the soliton. The latter results in a non-monotonic temperature dependence of the soliton energy at fixed finite density. At densities larger than the nuclear matter one ρ_{nm}, which are relevant for the heavy-ion experiments, the reduction of the soliton energy at finite temperature is much smaller than one at $T = 0$ (see fig. 12).

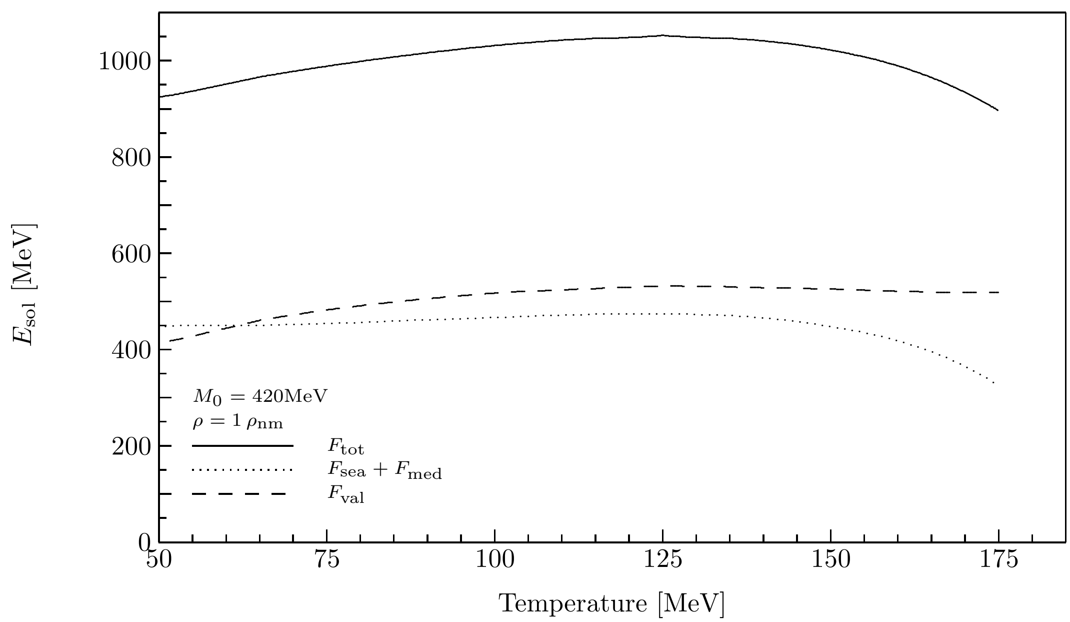

FIG. 13. Soliton energy and contributions coming from the valence quarks and from the polarized continua as a function of temperature at density $\rho = \rho_{\rm nm}$.

In particular, at ρ_{nm} and $T > 100$ MeV the reduction is about 15% whereas at $T = 0$ is two times larger. Further, at densities larger than two times ρ_{nm} the soliton exists only at intermediate temperatures 100 MeV $< T <$ 180 MeV. It means that according to the present model calculations, the soliton is more stable in the hot matter than in the cold one. In fact, it can be easily understood. Because of the gap in the quark spectrum, $2M$, the Dirac sea is much less affected by the temperature than the Fermi sea (positive-energy part of the spectrum). Disordering mainly the Fermi sea the attractive interaction between the medium and the valence quarks and Dirac sea, which destabilize the soliton, is diminished.

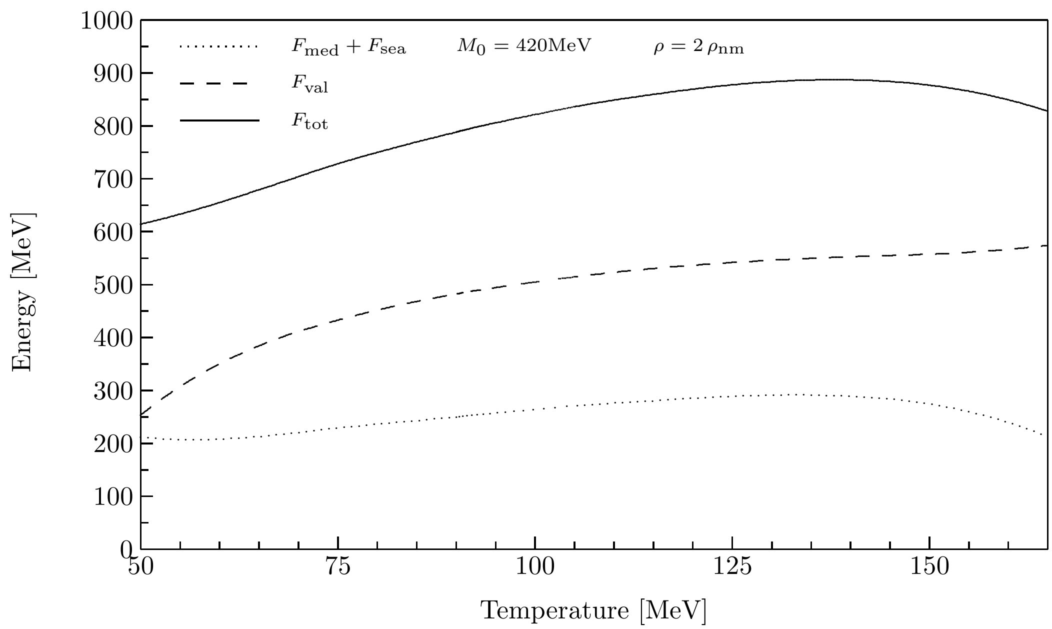

FIG. 14. Soliton energy and contributions coming from the valence quarks and from the polarized continua as a function of temperature at density $\rho = 2\,\rho_{\rm nm}$.

At temperatures close to the critical one, the thermal fluctuations become comparable with the chiral order parameter, the chiral condensate $\bar{q}q >$, they completely disorder the system and in particular, destroy the soliton.

It is interesting to look at the separated valence and sea contributions to the soliton energy presented in fig. 13 and 14 for two different medium densities, namely one and two times nuclear matter density, respectively. Whereas at low temperature almost a half of the energy comes from the polarized Dirac and Fermi sea, at increasing T the valence contribution becomes dominant. Also, in contrast to the valence quark contribution, which shows a strong dependence on T, the polarized sea contribution stays almost constant with T not close to the critical one but it is strongly affected by the medium density. In particular, going from one to two times ρ_{nm}, the sea contribution is reduced by a factor of two, which destabilizes the soliton. Further, at T close to the critical values it vanishes.

In order to illustrate the change of the soliton structure in hot medium we also plot the soliton square radius (42) as a function of density for different temperatures in figs. 15. All curves in fig. 15 show a clear trend to grow rapidly at temperatures close to the critical values which is an indication for delocalization of the soliton. At finite both density and temperature the radius is smaller than in the case of cold matter which is a sign for a stabilization of the soliton in hot medium compared to the case of cold one. At densities larger than two times ρ_{nm} the soliton exists only at intermediate temperatures.

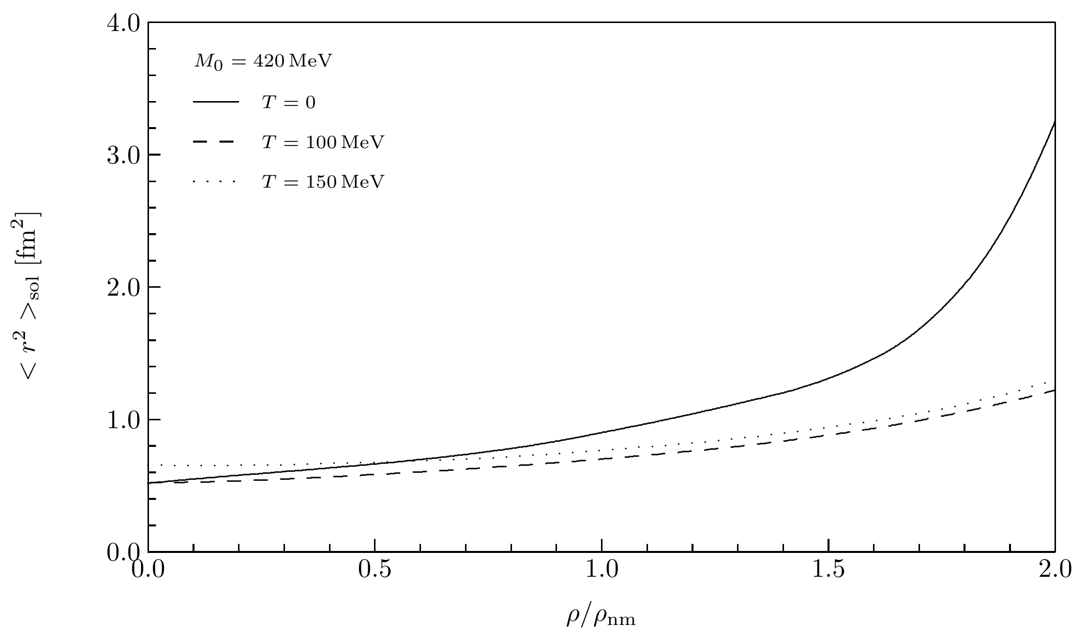

FIG. 15. Soliton m.s. radius as a function of density for vanishing as well as for finite temperature values.

B. In medium of nucleons

In this subsection we consider the $B = 1$ soliton in the hybrid model with N_c valence quarks interacting with both the quark Dirac sea and the nucleon Fermi sea via the meson fields. As in the quark medium the $B = 1$ soliton is obtained solving the Dirac equations (5),(31) together with meson equations of motion (38),(39) which contain the finite-temperature nucleon part (34), and the constraint $\rho_B = const$ (33) to fix the chemical potential μ^N in the self-consistent iterative procedure. The energy of the $B = 1$ soliton is given again by the change of the free energy when the N_c valence quarks are added to the system.

On fig.16 the temperature dependence of the calculated $B = 1$ soliton energy for the nuclear matter density is compared with those of the quark matter. The two curves have similar trends at intermediate values of T and start to deviate at T close to critical one. The soliton in the nucleon matter is more bound and less affected by the temperature. At some critical temperature of about 200 MeV the soliton disappears – delocalization of the soliton. In fact, at the same temperature in the hybrid model there is also the chiral phase transition from Goldstone to Wigner phase. The m.s. soliton radius is presented in fig.17. Close to the critical temperature it starts to grow but it is much less pronounced than in the case of quark matter.

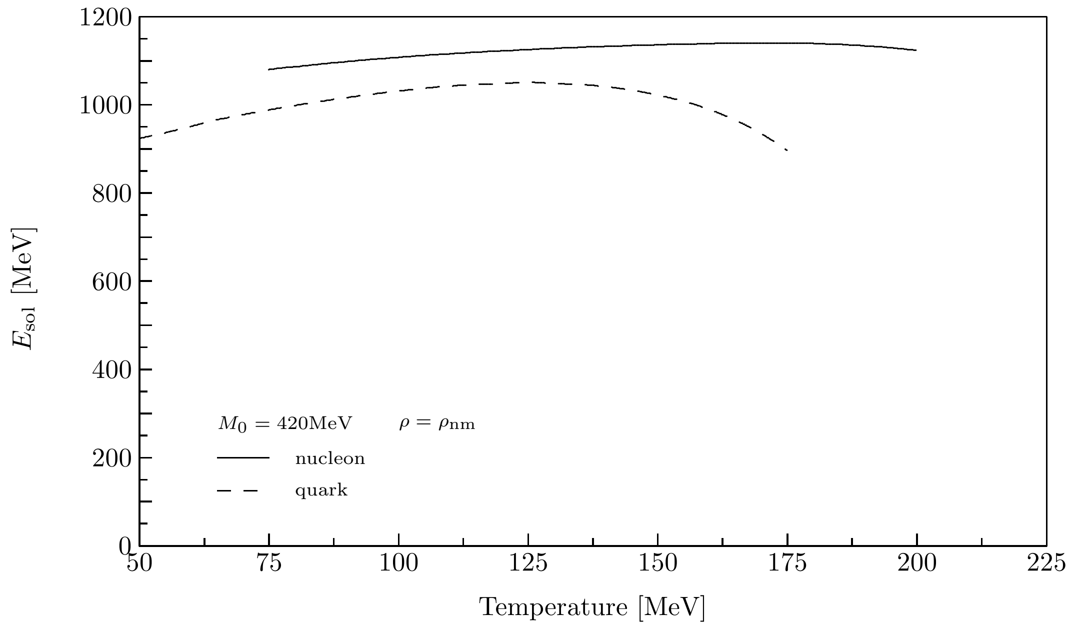

FIG. 16. Soliton energy in a nucleon matter compared to those in a quark matter for $\rho = \rho_{\rm nm}$.

Back to our discussion concerning the Walecka model, in our case this model picture is valid to some critical density of about two times nuclear matter density. At higher densities we found a delocalization transition to a quark matter. On fig.18 we also plot the coupling constant

$$g_N = E_{sol}/M \tag{43}$$

as a function of T at one nuclear matter density. As can be seen it stays almost constant which means that the relation (32) is a good approximation also in hot medium.

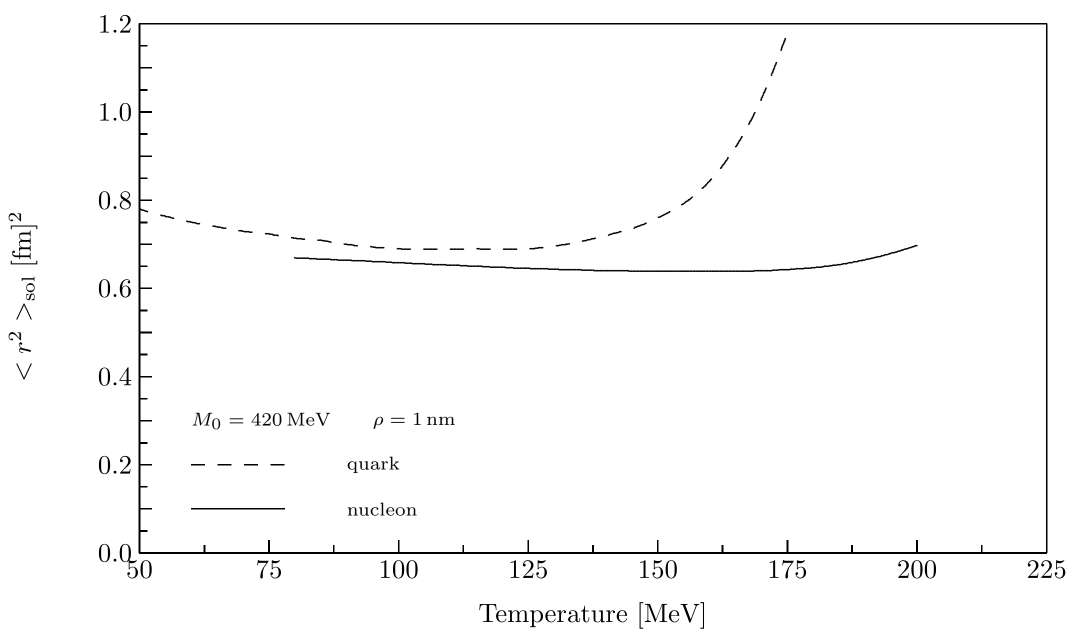

FIG. 17. Soliton m.s. radius in a nucleon matter compared to those in a quark matter for $\rho = \rho_{\rm nm}$.

In contrast to the quark matter the energy of the soliton in the nucleon matter is slightly above the threshold of $3M$. However, in the hybrid model this channel is physically closed by construction - before the phase transition, in the Goldstone phase, free quarks are not allowed to occupy the positive part of the quark spectrum.

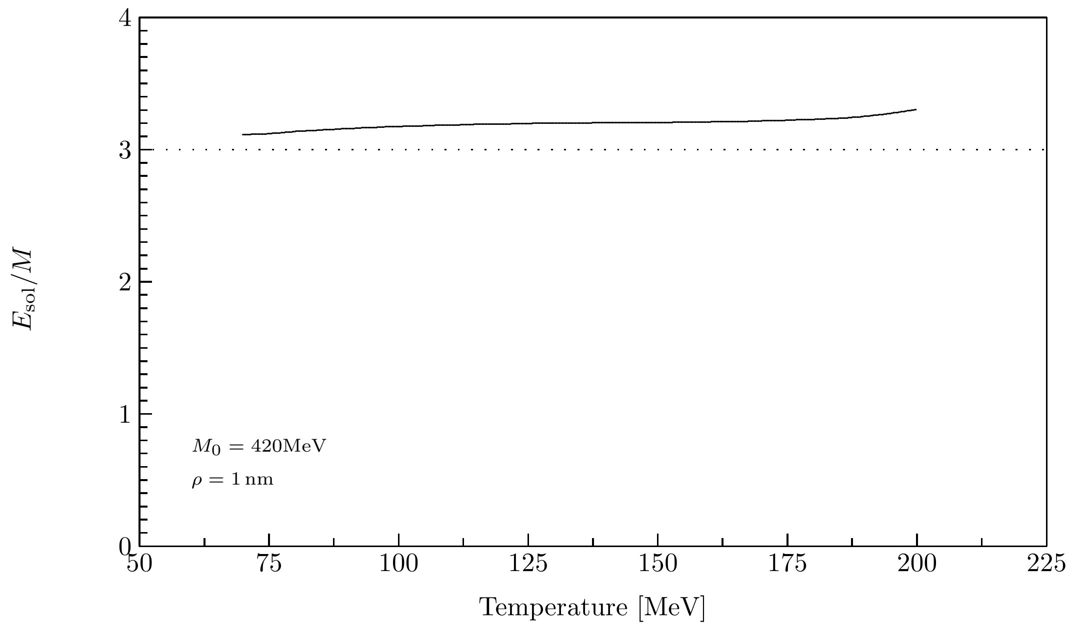

FIG. 18. Ratio E_{sol}/M in a nucleonic medium for $\rho = \rho_{\mathrm{nm}}$ as a function of temperature.

VI. SUMMARY

We study the bulk thermodynamical properties, some meson properties and the nucleon as a $B = 1$ soliton in hot medium using the NJL model in mean-field approximation. We consider the case of a quark medium in the NJL model as well as of a nucleon medium in a hybrid NJL model in which the quark Dirac sea is combined with a nucleon Fermi sea. In both cases we find that the chiral order parameter, the chiral condensate, gets modified and at some critical values of temperature and/or density vanishes which indicates a chiral phase transition from Goldstone to Wigner phase. At finite density the chiral order parameter and the constituent quark mass have a non-monotonic temperature dependence and at finite temperatures not close to the critical one they are less affected than in the cold matter. Below the chiral phase transition the pions are Goldstone bosons and their mass remains almost unchanged whereas the sigma mass follows the constituent quark mass. After the phase transition the mesons are parity-doubled and appear as broad resonances in the $\bar{q}q$ continuum. With increasing temperature the position of the resonance moves to higher energies and and its width increases very fast almost independently of the density. At some temperature of about 300 MeV the meson structure is very weak and is almost washed out. Whereas at low density and temperature values the quark matter and the nucleon matter provide similar results, at larger temperatures they differ significantly. In particular, the quark matter is much softer against thermal fluctuations and the chiral phase transition is rather smooth. The nucleon matter is much less affected by the temperatures even for values close to the critical one. The chiral phase transition is rather sharp and all thermodynamical variables show a large discontinuities which is an indication for a first order phase transition. We study also the structure of the baryon number one soliton of the NJL model immersed in a hot quark as well as in a nucleon medium. The polarization of both the Fermi and the Dirac sea is taken into account in a consistent way. We find that at finite density the temperature stabilizes the soliton and after some critical density it exists only at intermediate temperature values. In general, at finite temperature the soliton is more bound and less swelled than in the case of a cold matter. At some critical temperature we find no more a localized solution. In the case of nucleon matter this delocalization means a transition from the nucleon to the quark matter. The critical temperature coincides with those for the chiral phase transition. According to this model scenario one should expects at some critical temperature a common sharp phase transition from the nucleon to the quark matter with a restoration of chiral symmetry. All present results are obtained in mean-field (large N_c) approximation which means that the meson quantum (loop) effects are not included. However, it is expected [11,12]

that these effects would play a minor role near the phase transition and hence it would not change principally our results.

Acknowledgements: The work has been supported partially by the Bundesministerium für Bildung und Wissenschaft and the Bulgarian Science Foundation under contract Φ–32.

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[1] for reviews, see M.Fukugita, *Nucl.Phys.* **B4**(1988)105c; *Nucl.Phys.* **B9**(1989)291c; A.Ukawa,*Nucl.Phys.* **A10**(1989)66c; *Nucl.Phys.***A498**(1989)227c; N.H.Christ, *Nucl.Phys.***A544**(1991)81c; C.De Tar, Proceedings Lattice'94, eds. F.Karsch, J.Engels, E.Laermann and B.Peterson, Amsterdam, North-Holland,1995; Y.Iwasaki, in the same Proceedings

[2] F.Karsch, *Nucl.Phys.* **A590**(1995)367c

[3] Proceedings of Quark Matter93, *Nucl.Phys.* **A566**(1994); Proceedings of Quark Matter95, *Nucl.Phys.* **A590**(1995)

[4] Y.Nambu and G.Jona-Lasinio, *Phys.Rev.* **122** (1961) 354

[5] T.Hatsuda and T.Kunihiro, *Phys.Rep.* **247** (1994) 221

[6] J.Gasser and H.Leutwyler, *Phys.Lett.***B184** (1987) 83; *Phys.Lett.***B188** (1987) 477; *Phys.Lett.* **B184** (1987) 83; P.Gerber and H.Leutwyler,*Phys.Lett.***B321**(1989)387

[7] V.Bernard, Ulf-G.Meissner and I.Zahed, *Phys.Rev.* **D36** (1987) 819

[8] M.Asakawa and K.Yazaki, *Nucl.Phys.***A504** (1989)668

[9] Chr.V.Christov, E.Ruiz Arriola and K.Goeke, lecture at XXX Cracow School of Theoretical Physics, June 2-12, 1990, Zakopane, Poland, *Acta Phys.Polonica***22**(1991)187

[10] M.Lutz, S.Klimt and W.Weise, *Nucl.Phys.***A542** (1992)521

[11] J. Hüfner, S.P. Klevansky, E. Quack, P. Zhuang, *Phys.Lett.***B337**(1994)30;

[12] P. Zhuang, J. Hüfner, S.P. Klevansky, *Nucl.Phys.***A576**(1994)525;

[13] P. Rehberg, S.P. Klevansky, J. Hüfner, Preprint Heidelberg, HD-TVP-95. June 1995

[14] Chr.V.Christov and K.Goeke, *Nucl.Phys.* **A564**(1993)551

[15] T.Eguchi, H.Sugawara, *Phys.Rev.* **D10** (1974) 4257; T.Eguchi, *Phys.Rev.* **D14** (1976) 2755

[16] C.Bernard, *Phys.Rev.* **D9**(1974)3312; L.Dolan and R.Jackiw, ibid.**D9**(1974)3320

[17] M.Jaminon, G.Ripka and P.Stassart, *Nucl.Phys.* **A504**(1989)733

[18] C.Schüren, F.Döring, E.Ruiz Arriola and K.Goeke, *Nucl.Phys.***A565** (1993) 687

[19] Chr.V.Christov, A. Blotz, H.-C. Kim, P. V. Pobylitsa, T. Watabe, Th. Meissner, E. Ruiz Arriola and K. Goeke, *Prog.Part.Nucl.Phys.*, **Vol.37** (1996)1

[20] S.Kahana and G.Ripka, *Nucl.Phys.***A429** (1984) 462

[21] D.I.Diakonov, V.Yu.Petrov and P.V.Pobylitsa, *Nucl.Phys.***B306**(1988)809

[22] H.Reinhardt and R.Wünsch, *Phys.Lett.* **215B**(1988)577; ibid. **B230** (1989) 93

[23] T.Meissner, F.Grümmer and K.Goeke, Phys.Lett. **227B** (1989) 296; *Ann.Phys.* **202** (1990) 297; T.Meissner and K.Goeke, *Nucl.Phys.* **A254** (1991) 719

[24] V.Bernard and Ulf-G.Meissner, *Phys.Lett.* **B227** (1989)465; *Ann.Phys.* **206**(1991)50

[25] B.D.Serot and J.D.Walecka, *Advance in Physics***16** (1986)(Plenum Press)